Pose Correction by Space-Time Integration

Supplementary Material

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Let a RBF linear system

$$\mathbf{A} = \begin{pmatrix} \Phi_{0,0} & \Pi_0 \\ \Pi_0^{\mathrm{T}} & \mathbf{0} \end{pmatrix}$$

and its LU factorization (with partial pivoting) be given by $\mathbf{A} = \mathbf{P}^{\mathrm{T}}\mathbf{L}\mathbf{U}\mathbf{R}$. The matrices $\mathbf{P}$ and $\mathbf{R}$ are permutations ($\mathbf{R} = \mathbf{I}$ in the first iteration), $\mathbf{L}$ and $\mathbf{U}$ are lower and upper triangular.

The goal of factorization update is to reuse this known factorization for the derivation of the LU factorization of an RBF linear system that is extended by m new RBF centers and which has the form

$$\mathbf{A}' = \begin{pmatrix} \Phi_{0,0} & \Phi_{1,0} & \Pi_0 \\ \Phi_{1,0}^{\mathrm{T}} & \Phi_{1,1} & \Pi_1 \\ \Pi_0^{\mathrm{T}} & \Pi_1^{\mathrm{T}} & \mathbf{0} \end{pmatrix} .$$

We rewrite $\mathbf{A}'$ by first applying global row and column permutations $\mathbf{G}$ to the new system matrix $\mathbf{A}'$ such that we can reuse the factorization of $\mathbf{A}$:

$$\mathbf{A}' = \mathbf{G} \begin{pmatrix} \Phi_{0,0} & \Pi_0 & \Phi_{1,0} \\ \Pi_0^{\mathrm{T}} & \mathbf{0} & \Pi_1^{\mathrm{T}} \\ \Phi_{1,0}^{\mathrm{T}} & \Pi_1 & \Phi_{1,1} \end{pmatrix} \mathbf{G} = \mathbf{G} \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{C}^{\mathrm{T}} & \Phi_{1,1} \end{pmatrix} \mathbf{G} .$$

The permutation $\mathbf{G}$ swaps the last two rows and columns of block matrices, respectively. We did also substitute the block matrix $\mathbf{C}^{\mathrm{T}} = \begin{pmatrix} \Phi_{1,0}^{\mathrm{T}} & \Pi_1 \end{pmatrix}$.

It is now possible to derive the updated LU factorization of $\mathbf{A}' = \mathbf{P}'^{\mathrm{T}}\mathbf{L}'\mathbf{U}'\mathbf{R}'$ by using only

identity transformations:

$$
\begin{aligned}
\mathbf{A}' = \mathbf{G} &\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{0} & \Phi_{1,1}-\mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{C} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{L}^{-1}\mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{C} \\ \mathbf{0} & \Phi_{1,1}-\mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{C} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{C}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{U}^{-1} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{U}\mathbf{R} & \mathbf{L}^{-1}\mathbf{P}\mathbf{C} \\ \mathbf{0} & \Phi_{1,1}-\mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{R}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{S} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{U} & \mathbf{T} \\ \mathbf{0} & \check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}}\tilde{\mathbf{U}} \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{S} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}} \end{pmatrix} \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{L}}^{-1}\check{\mathbf{P}} \end{pmatrix} \begin{pmatrix} \mathbf{U} & \mathbf{T} \\ \mathbf{0} & \check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}}\tilde{\mathbf{U}} \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{S} & \check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}} \end{pmatrix} \begin{pmatrix} \mathbf{U} & \mathbf{T} \\ \mathbf{0} & \tilde{\mathbf{U}} \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\begin{pmatrix} \mathbf{P}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{P}}^{\mathrm{T}} \end{pmatrix} \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{P}} \end{pmatrix} \begin{pmatrix} \mathbf{P}^{\mathrm{T}}\mathbf{L} & \mathbf{0} \\ \mathbf{S} & \check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}} \end{pmatrix} \begin{pmatrix} \mathbf{U} & \mathbf{T} \\ \mathbf{0} & \tilde{\mathbf{U}} \end{pmatrix} \begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix} \mathbf{G} \\
= \mathbf{G} &\underbrace{\begin{pmatrix} \mathbf{P}^{\mathrm{T}} & \mathbf{0} \\ \mathbf{0} & \check{\mathbf{P}}^{\mathrm{T}} \end{pmatrix}}_{\mathbf{P}'^{\mathrm{T}}} \underbrace{\begin{pmatrix} \mathbf{L} & \mathbf{0} \\ \check{\mathbf{P}}\mathbf{S} & \tilde{\mathbf{L}} \end{pmatrix}}_{\mathbf{L}'} \underbrace{\begin{pmatrix} \mathbf{U} & \mathbf{T} \\ \mathbf{0} & \tilde{\mathbf{U}} \end{pmatrix}}_{\mathbf{U}'} \underbrace{\begin{pmatrix} \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{pmatrix}}_{\mathbf{R}'} \mathbf{G} \; .
\end{aligned}
$$

In this derivation we have made the substitutions $\mathbf{S}=\mathbf{C}^{\mathrm{T}}\mathbf{R}^{\mathrm{T}}\mathbf{U}^{-1}$ and $\mathbf{T}=\mathbf{L}^{-1}\mathbf{P}\mathbf{C}$ that require two triangular back-substitutions for known triangular matrices. Additionally, we substituted the $m \times m$ LU factorization with partial pivoting of the matrix $\Phi_{1,1}-\mathbf{C}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{C}=\Phi_{1,1}-\mathbf{S}\mathbf{T}$, which we denoted by $\check{\mathbf{P}}^{\mathrm{T}}\tilde{\mathbf{L}}\tilde{\mathbf{U}}$.